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**Title : Acceleration and Special Relativity.**

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## **ACCELERATION AND SPECIAL RELATIVITY**

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### **ABSTRACT**

The integration of acceleration over time before reaching the uniform velocity turns out to be the source of all the special relativity effects. It explains physical phenomena like clocks comparisons. The equations for space-time, mass and energy are presented. This phenomenon complements the explanation for the twins' paradox. A Universal inertial frame is obtained.

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## I. INTRODUCTION

The basic equations of the special relativity concern time measurement in different moving frames. Assume the velocity of a moving frame is  $v$ ,  $t$  is the time in the inertial frame and  $t'$  is the time in the moving reference frame,  $c$  is the velocity of light. The coordinates in the inertial and the reference frame are  $x$  and  $x'$  respectively. The equations according to special relativity are:

$$m_2 = \frac{m_1}{\sqrt{1 - \frac{u^2}{c^2}}}$$

[1]

The rate of time increment  $dt'$  at the moving clock compared to the rate  $dt$  at rest in the inertial frame, where  $x'$  is constant, so  $dx'=0$ , and

$$dt = \frac{dt'}{\sqrt{1 - \frac{V^2}{c^2}}}$$

[2]

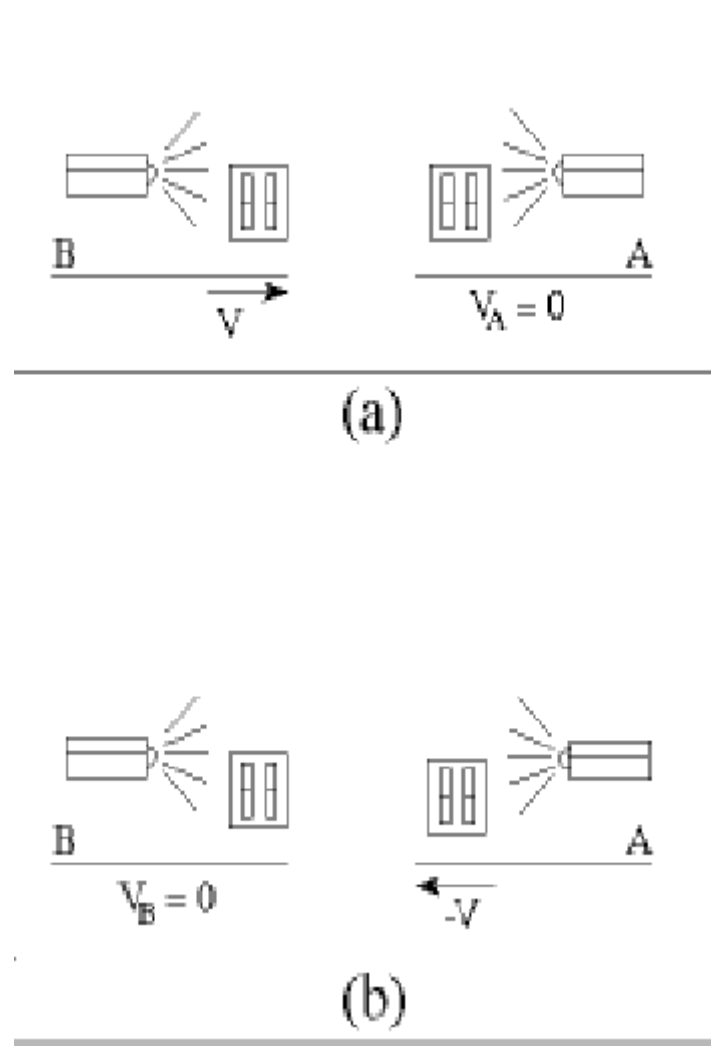


FIG. 1. (a) Clock A and a video camera at rest in the inertial frame.  
 Clock B and a video camera move with velocity  $V$  towards clock A.  
 (b) Clock A moves with velocity  $-v$  relative to clock B and a video camera which are at rest in B's reference frame.

Consider a clock, which has a digit display of a fraction of a second. One clock of this type is in the inertial frame, clock A. The second clock B is in the reference frame as illustrated in Fig. 1. In each frame there is a video camera which records the change in the clock digits. While clock B is approaching clock A, the camera in the reference frame records clocks A and B in the picture. From A observation, A is at rest and B moves with velocity V, and the time interval is

$$dt_A = \frac{dt_B}{\sqrt{1 - (V^2/C^2)}} .$$

The clock B digits are changing slower than clock A digits. From B observation A moves with velocity - V and B is at rest, the time interval is

$$dt_B = \frac{dt_A}{\sqrt{1 - \frac{V^2}{C^2}}}$$

Clock A digits change slower. There is contradiction between the two video records, which record the same physical event. The solution to the contradiction comes from the discovery that there is a difference between the two frames. Only one of the frames accelerated to reach velocity V. The following term is represented:

$$V = \int_0^{t_1} a(t) dt$$

[3]

a(t) is the time dependent acceleration, t<sub>1</sub> is the time until the body reaches

velocity  $V$  and stops accelerating. The clock's frame that is considered to move with velocity  $V$  and has the relativistic effects, is the clock that accelerated from rest to velocity  $V$  and continues to move with uniform velocity  $V$  afterwards. The special relativity effects are derived from the acceleration which gets the system to the uniform velocity  $V$ . This is a different phenomenon than the acceleration influence on relativistic effects at the moment of acceleration [1], which is presented as the usual explanation to the twins paradox.

The twins' paradox considers two twins, one is flying to space and returns to earth, the other stays on earth. The one who flew to space aged less. The explanation [2] is that the twin who flew into space felt acceleration so his time changed slower. This is correct, but not only the magnitude of the acceleration at the moment it acts affects the difference in time, this moment can be infinitesimal and its influence can be infinitesimal. The magnitude of the phenomenon relate to the integration of the acceleration over the time of the acceleration And the total difference in time relate to the period of time the object continued to move with the uniform velocity it reached after it stopped accelerating

If we take two planes with identical acceleration periods and magnitudes, the time in the one that has flown a longer period of time before landing, will be retarded compared to the other [3,4].

## II. MORE PHYSICAL DESCRIPTIONS

Consider two clocks, A and B which are placed on coordinates 0 and  $X_1$  respectively on the x axis in the inertial frame. Clocks A and B are Synchronized with each other. Clock C moves with velocity  $V$  in this frame. Clock C passes clock A at  $t=0$  and  $x=0$ . At this point, clock C sets its time to be  $t'=0$ , so  $x'=0$ . When C is passing clock B, a camera in B's inertial frame photographs both clock B and C, and a camera in C's frame photograph both clock B and C.

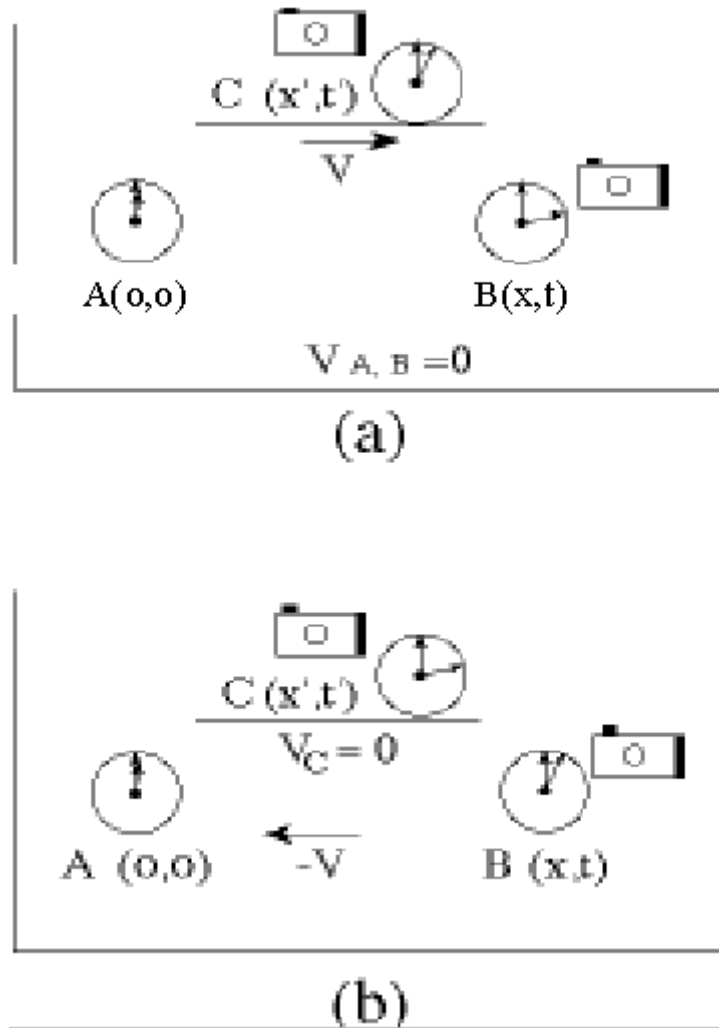


FIG.2. (a) Clocks A and B at rest in the inertial frame, clock C moving with velocity  $v$ , camera D is at rest in the inertial Frame. (b) Clock C is at rest in its reference frame, clocks A and B move with velocity  $-V$  relative to clock C. Camera E is at rest in clock C reference frame.

When clock C passes clock B, they would not be synchronized. Clock B indicators will show time  $t$ , clock C indicators will show time  $t'$ .



$$\begin{aligned}
 t' &= \frac{t - \frac{VX}{c^2}}{\sqrt{1 - \frac{V^2}{c^2}}} = \frac{t - \frac{V^2 t}{c^2}}{\sqrt{1 - \frac{V^2}{c^2}}} \\
 &= t \sqrt{1 - \frac{V^2}{c^2}}
 \end{aligned}$$

[4]

where  $X=Vt$ , and therefore  $t'<t$ . The calculation of time where C is considered to be in the inertial frame and A and B move with velocity - V compared to C is

$$\begin{aligned}
 t &= \frac{t' + (V/C^2)x'}{\sqrt{1 - (V^2/C^2)}} & X' &= -Vt \\
 t &= \sqrt{1 - (V^2/C^2)} t'
 \end{aligned}$$

[5]

We have a contradiction between equations 4 and 5. We would have two pictures of the same physical state but the clock indicator positions would be different in the pictures.

We substitute our solution to the contradiction, eq. 3, to the standard special relativity equations and we get the modified equations:

$$\begin{aligned}
 x' &= \frac{x - \left( \int_0^{t_1} a(t) dt \right) t}{\sqrt{1 - \left( \frac{\int_0^{t_1} a(t) dt}{C} \right)^2}} \\
 y' &= y \\
 z' &= z \\
 t' &= \frac{t - \left( \int_0^{t_1} a(t) dt \right) X/C^2}{\sqrt{1 - \left( \frac{\int_0^{t_1} a(t) dt}{C} \right)^2}}
 \end{aligned}$$

[6]

$$x = x' \sqrt{1 - \left( \frac{\int_0^{t_1} a(t) dt}{C} \right)^2} + \left( \int_0^{t_1} a(t) dt \right) \cdot t$$

$$t = t' \sqrt{1 - \left( \frac{\int_0^{t_1} a(t) dt}{C} \right)^2} + \left( \int_0^{t_1} a(t) dt \right) t \cdot \frac{x}{C^2}$$

[7]

Equations [6,7] can be obtained in more rigorous way by substituting eq. 3, on the Lorentz coordinates transformations equations.

Equations [6] are identical to Eq. [7] because the inertial frame is always the frame which has not accelerated and the frame which has relativistic effects is the frame which accelerated to velocity V.

We get a symmetry breaking in special relativity. The equations of the standard Special relativity, for events observed in S' frame with relative velocity, exchanged from V to - V do not describe the physical reality.

The equations of the addition of velocities:

S' frame moves with velocity V relative to S and an object A moves with velocity u' relative to S', which accelerated from rest in S' frame at t<sub>2</sub> and stopped accelerating at t<sub>3</sub>.

Let x',t' be the coordinates in S' frame

$$x' = \frac{x - \int_0^{t_1} a(t) dt \cdot t}{\sqrt{1 - \left( \frac{\int_0^{t_1} a(t) dt \cdot t}{C} \right)^2}} = \int_{t_2}^{t_3} a(t) dt \cdot t'$$

$$t' = \frac{t - \int_{t_2}^{t_3} a(t) dt \cdot t \frac{X}{C^2}}{\sqrt{1 - \left( \frac{\int_0^{t_1} a(t) dt \cdot t}{C} \right)^2}} \quad [8]$$

$$x = \frac{\int_{t_2}^{t_3} a(t) dt + \int_0^{t_1} a(t) dt}{1 + \int_{t_2}^{t_3} a(t) dt \cdot \int_0^{t_1} a(t) dt / C^2} t$$

$$u = \frac{\int_{t_2}^{t_3} a(t) dt + \int_{t_0}^{t_1} a(t) dt}{1 + \int_{t_2}^{t_3} a(t) dt \cdot \int_0^{t_1} a(t) dt / C^2}$$

[9]

u is the velocity of object A in the inertial frame s.

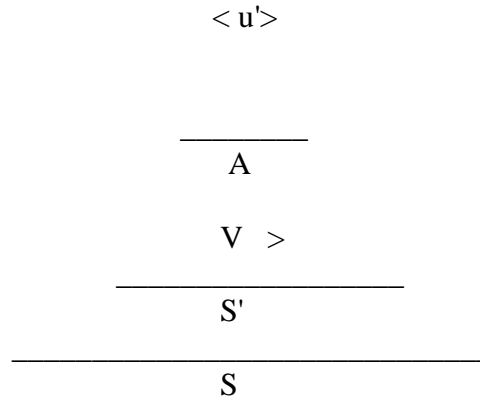


Fig 3: S' moves with velocity  $V$  relative to the inertial frame  $S$ .  
Object  $A$  moves with velocity  $u'$  relative to  $S'$ .

Mass transformation:  $m_0$  is the rest mass.  $m$  is the mass which accelerated to velocity  $V$ .

$$m = \frac{m_0}{\sqrt{1 - \left( \frac{\int_0^{t_1} a(t) dt}{C} \right)^2}}$$

[10]

Let  $m_1$  be a mass at rest in reference  $s'$  which moves with velocity  $V$ , compared to inertial frame  $s$ . Assume mass  $m_1$  accelerated to velocity  $u'$  relative to frame  $s'$ .

There are two options for moving with velocity  $u'$  relative to  $s'$ . One moving with velocity  $u'$ , the other with velocity  $-u'$ .

For the standard special relativity equations we have

$$m_2 = \frac{m_1}{\sqrt{1 - \frac{u^2}{C^2}}}$$

$m_2$  the mass moving with velocity  $|u'|$  relative to  $s'$ . But according to the postulate that the relativistic effects depend on the acceleration that gets the object to velocity  $V$ , we must consider the velocity compared to the inertial frame  $s$ .

For velocity equals  $u'$

$$u = \frac{\int_{t_2}^{t_3} a(t)dt + \int_0^{t_1} a(t)dt}{1 + \int_{t_2}^{t_3} a(t)dt \cdot \int_0^{t_1} a(t)dt / C^2} \quad (i)$$

[11]

$u$  is the velocity compared to the inertial frame  $s$ ,  $u' = \int_{t_2}^{t_3} a(t)dt$   $V = \int_0^{t_1} a(t)dt$  .

For velocity equals  $-u'$

$$u = \frac{-\int_{t_2}^{t_3} a(t)dt + \int_0^{t_1} a(t)dt}{1 - \int_{t_2}^{t_3} a(t)dt \cdot \int_0^{t_1} a(t)dt} \quad (ii)$$

$$m_2 = \frac{m_0}{\sqrt{1 - \frac{u^2}{C^2}}} \quad m_1 = \frac{m_0}{\sqrt{1 - \left( \frac{\int_0^{t_1} a(t) dt}{C} \right)^2}}$$

$$m_2 = \frac{m_1 \sqrt{1 - \left( \frac{\int_0^{t_1} a(t) dt}{C} \right)^2}}{\sqrt{1 - \frac{u^2}{C^2}}}$$

[12]

Since  $m_2$  is dependent on the acceleration from 0 velocity, it has different values if the velocity compares to frame  $s'$  is  $u'$  or  $-u'$ . Assume mass  $m$  is accelerated to velocity  $V$  relative to earth first in the direction of earth's rotation second to velocity  $V$  relative earth in opposite direction to earth's rotation. The mass will be different in the two cases. For  $u'=V$  we get

$$u = \frac{2V}{1 + \frac{V^2}{C^2}}$$

For  $u' = -V$ ,  $u=0$  and  $m_2=m_0 < m_1$ .

When  $u < -V$ ,  $m_2 > m_0$

If we would check mass in continuity of velocities and in different directions, we would find the universal inertial frame. The measurement rate of an atomic clock at different speeds and in different directions, gives different clock rates. When the rate is the highest, after eliminating the effects of the earth non-inertial frame, we will get the universal inertial frame.

$$dt = \frac{dt_0}{\sqrt{1 - \left( \frac{\int_0^{t_1} a(t) dt}{C} \right)^2}}$$

[13]

Assuming  $u = \int_0^{t_1} a(t) dt$

is the velocity relative to the universal inertial frame,  $dt_0$  is the rate of the moving clock,  $dt$  is the rate of the clock at rest. For  $v > u'$  while  $v$  is the velocity of earth's relative to the universal inertial frame,  $dt_0$  will be higher than  $dt_1$ , the clock rate at earth.

The best place to find the universal inertial frame is in space where the influence of the rotational movement of earth is reduced.

The total energy equation is

$$E = \frac{E_0}{\sqrt{1 - \left( \frac{\int_0^{t_1} a(t) dt}{C} \right)^2}}$$

[14]



### **III. Experiment results supported the universal inertial frame**

Hafele and Keating experiment [4] strongly support the existence of a Universal inertial frame. In this experiment, a time of a flying around the world clock compared to a clock that stayed on a certain position on the surface of earth. The measurements was repeated with the flying clock circling the earth in the eastward direction , and in the westward direction.

The results of the experiment showed that the flying clock's time lag in comparison to the clock on the surface during the eastward trip and it's time was early, in comparison to the clock on the surface during the westward trip. The theoretical calculations[3] took the axis of the north pole as an inertial frame this gave a good approximation to the experience results .However the experiment showed no symmetry in the mean errors between the experience and the predicted results at the eastward and westward directions. For the westward direction the mean difference between the predicted and the experiment results is 0.7% and the mean difference in the eastward direction is 15%. This imply the inertial frame considered is not the exact inertial frame . Although Hafele and Keating predicted calculations gave a good approximation to the experience result by using General Relativity in the calculations. And not only conventional special relativity, which gives completely different results from the experiment. Calculations of the exact inertial frame should take into account the Universal inertial frame.

The experiment was made four times at each direction, part of the results were out

of range of the predicted measurements errors. This is because the earth rotates around itself and around the sun so that the clocks directions relative to the universal inertial frame was changing.

In summary, the relativistic effects of special relativity are derived from integration of the acceleration over time. This complements the full explanation on the twins paradox. A universal reference frame is obtained. It implies that on a proper direction there is a range of velocity in which the rate of time on a moving frame can increase, the magnitude of mass can decrease. While in the standard special relativity the time rate of a moving frame can only decrease compared to rate on earth and the magnitude of mass can only increase. . An experiment based on the rate's measurement of atomic clock as we described could discover the speed and the direction the Earth moves compared to the Universe before the Big Bang. To those which are not convinced in the theory of the Big Bang, a future experiment for measuring the Universal inertial frame in several locations in the Universe, could determine if the Universe really expanded from one location. To find at which location the Big Bang occurred, the measurement should be made in at least two places. The intersection of the two directions obtained from the measurements of the Universal inertial frames will determine the location of the Big Bang.

If the Universe expands from one location, rate measurement of an atomic clock made of matter from one place in the Universe when compared to the rate of an atomic clock made of matter from another place in the Universe, would show that

the highest rate of both clocks when measured at the same place is obtained at the same direction and velocity, when both matters come from the same origin. If the result shows two different directions or velocities, then there are two origins.

## REFERENCES

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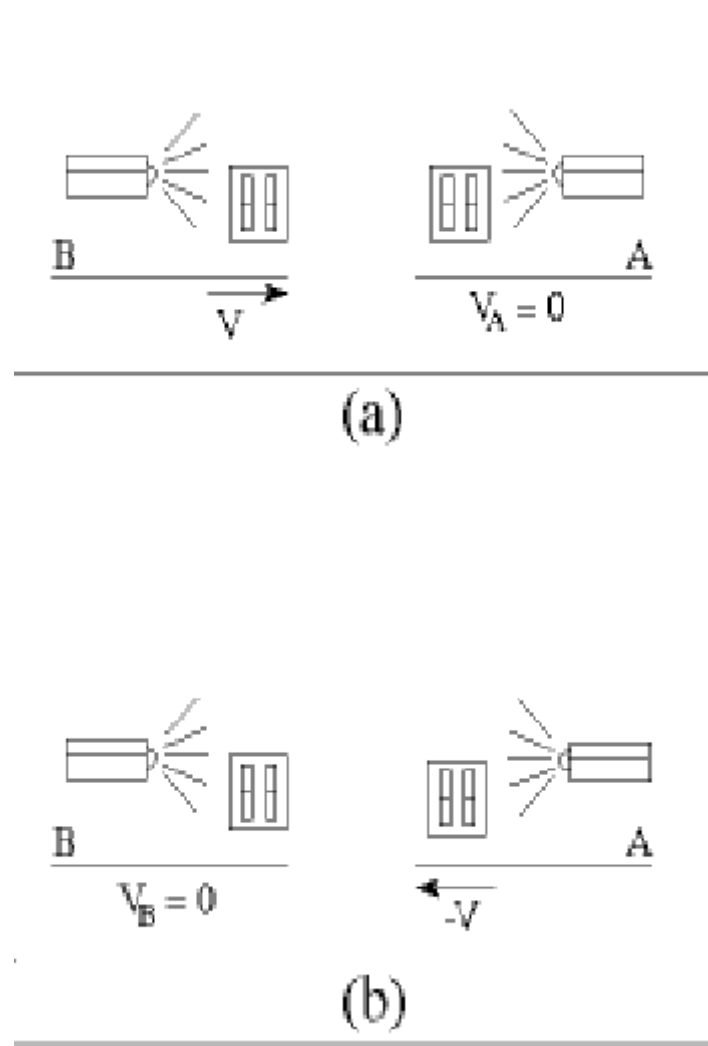


Fig.1

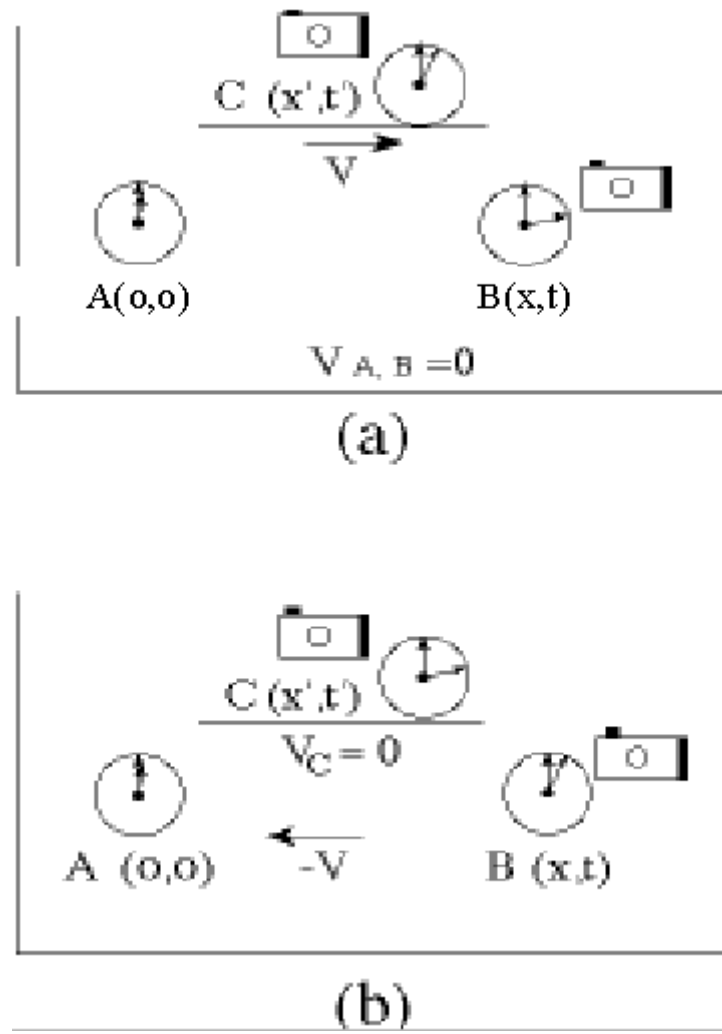


Fig.2